

Generalized hadron structure and elastic scattering

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Abstract. The new parameterization of the Generalized Parton Distributions (GPDs) t -dependence is investigated. It is shown that the new form of the GPDs allows one to reproduce sufficiently well the electromagnetic form factors of the proton and neutron at small and large momentum transfer and obtain a good description of the elastic nucleon-nucleon scattering at high energies.

1 Introduction

The new experimental data of the LHC, especially of the TOTEM Collaborations [1,2], require deep understanding of hadron interactions at high energies which are related with inner structure of the hadrons [3]. The electromagnetic current of a nucleon is

$$\begin{aligned} J_\mu(P', s'; P, s) &= \bar{u}(P', s') A_\mu(q, P) u(P, s) \\ &= \bar{u}(P', s') (\gamma_\mu F_1(q^2)) + \frac{1}{2M} i\sigma_{\mu\nu} q_\nu F_2(q^2) u(P, s), \end{aligned} \quad (1)$$

where $P, s, (P', s')$ are the four-momentum and polarization of the incoming (outgoing) nucleon and $q = P' - P$ is the momentum transfer. The quantity $A_\mu(q, P)$ is the nucleon-photon vertex.

The Sachs form factors [4] are related with the Dirac and Pauli form factors

$$G_E^p(t) = F_1^p(t) + \frac{t}{4M^2} F_2^p(t); \quad G_M^p(t) = F_1^p(t) + F_2^p(t); \quad (2)$$

with $(t = -q^2 < 0)$. Their three-dimensional Fourier transform provides the electric-charge-density and the magnetic-current-density distribution [5]. Normalization requires $G_E^p(0) = 1$, $G_E^n(0) = 0$ corresponding to proton and neutron electric charges; $G_M(0) = (G_E(0) + k) = \mu$ defines the proton and neutron magnetic moments. Here $\mu_p = (1 + 1.79) \frac{e}{2M}$ is the proton magnetic moment and $k = F_2(0)$ is the anomalous magnetic moment: $k_p = 1.79$.

The experiments based on the Rosenbluth separation method [6],

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_{\text{Mott}}}{\epsilon(1 + \tau)} [\tau G_M^2(t) + \epsilon G_E^2(t)] \quad (3)$$

where $\tau = Q^2/4M_p^2$ and $\epsilon = [1 + 2(1 + \tau) \tan^2(\theta_e/2)]^{-1}$ is a measure of the virtual photon polarization, suggested that the scaling behavior of both the proton form factors approximately are described by the dipole form

$$G_E^p \approx \frac{G_M^p}{\mu_p} \approx \frac{G_M^n}{\mu_n} \approx G_D = \frac{\Lambda^4}{(\Lambda^2 - t)^2}, \quad (4)$$

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which leads to

$$F_1^D(t) = \frac{4M_p^2 - t\mu_p}{4M_p^2 - t} G_D; \quad F_2^D(t) = \frac{4k_p M_p^2}{4M_p^2 - t} G_D; \quad (5)$$

with $\Lambda^2 = 0.71 \text{ GeV}^2$.

The information about the form-factors can also be obtained by using the polarization method [7,8]. Using the longitudinal components of the recoil proton polarization in the electron scattering plane, the ratio of the form factors

$$\frac{G_E^p}{G_M^p} = -\frac{P_t}{P_l} \frac{E + E'}{2M_p} \tan(\theta/2) \quad (6)$$

can be obtained. These data manifested a strong deviation from the scaling law and disagreement with the data obtained by the Rosenbluth technique. The results consist in an almost linear decrease of G_E^p/G_M^p .

The electromagnetic form factors can be obtained from the first moments of the Generalized Parton Distributions (GPDs) [9,10,11,12]

$$F_1(t) = \sum_q \int_0^1 dx \mathcal{H}^q(x, t); \quad F_2(t) = \sum_q \int_0^1 dx \mathcal{E}^q(x, t), \quad (7)$$

following from the sum rules [10,11].

The Regge-like picture for GPDs was proposed in [13]

$$\mathcal{H}^q(x, t) \approx \frac{1}{x^{\alpha'} (1-x)^t} q(x). \quad (8)$$

In [14,15], it was shown that at large $x \rightarrow 1$ the behavior of GPDs requires a power of $(1-x)$

$$\mathcal{H}^q(x, t) \approx \exp[a (1-x)^n t] q(x). \quad (9)$$

with $n \geq 2$. It was noted that $n = 2$ naturally leads to the Drell–Yan–West duality between parton distributions at large x and the form factors.

2 Transfer dependence of General parton distributions

In [16], the new form of the momentum transfer dependence of the General Parton Distribution was proposed

$$\mathcal{H}^q(x, t) = q(x) \exp[a_+ t (1-x)^2/x^m]. \quad (10)$$

The value of the parameter $m = 0.4$ is fixed by the low t experimental data while the free parameters a_{\pm} (a_+ – for \mathcal{H} and a_- – for \mathcal{E}) were chosen to reproduce the experimental data. A large momentum transfer behavior corresponds to $x \approx 1$ in (10), where the dependence on m is weak. For the parton distribution we choose the PDFs obtained in [17].

Correspondingly, we obtain

$$\mathcal{E}^q(x, t) = \mathcal{E}^q(x) \exp[a_- t (1-x)^2/x^{0.4}]. \quad (11)$$

with

$$\mathcal{E}^u(x) = \frac{k_u}{N_u} (1-x)^{\kappa_1} u(x), \quad \mathcal{E}^d(x) = \frac{k_d}{N_d} (1-x)^{\kappa_2} d(x), \quad (12)$$

where $\kappa_1 = 1.53$ and $\kappa_2 = 0.31$ [13].

According to the normalization of the Sachs form factors $k_u = 1.673$, $k_d = -2.033$, $N_u = 1.53$, $N_d = 0.946$.

3 Proton and neutron electromagnetic form factors

The ratio of the Sachs proton form factors is shown in Fig. 1. Our theoretical curve coincides with the polarization data.

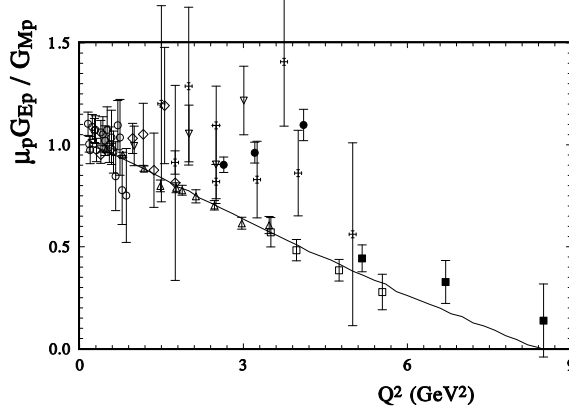


Fig. 1. $\mu_p G_E^p / G_M^p$ (the line corresponds to model calculation; the experimental data are from [18,19,20,21,22,23,24,25]).

It is to be noted, that some changing of the size of a_- can lead to the change of the slope of the ratio $\mu_p G_E^p / G_M^p$ and leads to the Rosenbluth data. However, the neutron data on the momentum dependence of the form factors do not allow this.

The isotopic invariance can be used to relate the proton and neutron GPDs. Hence, we do not change any parameter and keep the same t -dependence of GPDs as in the case of proton. As a result, we obtain the good description of the electromagnetic form factors of the neutron too.

4 Gravitational form factors of the nucleon

Taking the matrix elements of the energy-momentum tensor $T_{\mu\nu}$ [10]

$$\begin{aligned} \langle p' | \hat{T}_{\mu\nu}^{Q,G}(0) | p \rangle = & \bar{u}(p') \left[A^{Q,G}(t) \frac{\gamma_{\mu\nu} P_\nu}{2} + B^{Q,G}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{4M_N} \right. \\ & \left. + C^{Q,G}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{M_N} \pm \bar{c}(t) g_{\mu\nu} \right] u(p) \end{aligned} \quad (13)$$

one can obtain the gravitational form factors of quarks which are related to the second moments of GPDs. For $\xi = 0$ one has

$$A_q(t) = \sum_q \int_0^1 dx x \mathcal{H}_q(x, t); \quad B_q(t) = \sum_q \int_0^1 dx x \mathcal{E}_q(x, t). \quad (14)$$

This representation combined with our model allows one to calculate the gravitational form factors of valence quarks and their contribution (being just their sum) to the gravitational form factors of the nucleon. Our results for $A_{u+d}(t)$ are shown in Fig. 2 a (for small momentum transfer) and in Fig. 2 b (for large momentum transfer). We can describe the obtained form of $A_{Gr.}(t)$ by the standard dipole form but with a sufficiently large size of $\Lambda^2 = 1.8 - 2.0 \text{ GeV}^2$. The corresponding results are shown in Fig. 2 a,b.

Of course, the dipole presentation of $A_{Gr.}(t)$ is the simplest form close to the standard form of the electromagnetic form factors. A slightly more complicated form can be taken as

$$A_{Gr.}(t) = 1 / [1 + q/\Lambda_1 - t/\Lambda_2]^2, \quad (15)$$

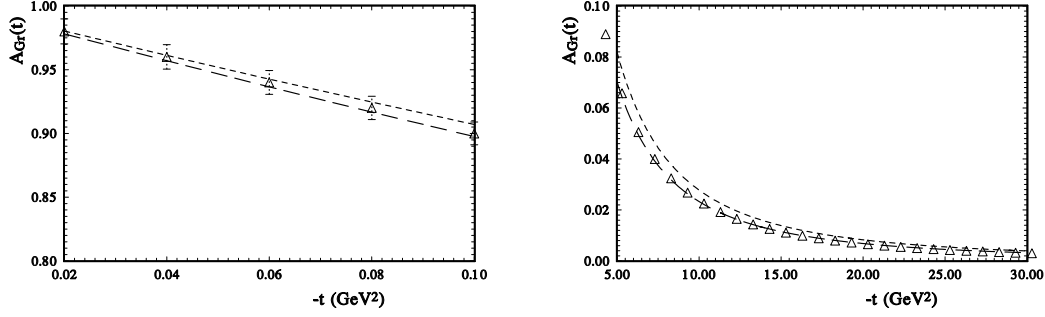


Fig. 2. $A_{Gr,t}$ (hard and dot-dashed lines correspond to the variant with $\Lambda^2 = 2.0 \text{ GeV}^2$ and $\Lambda^2 = 1.8 \text{ GeV}^2$; triangles are the integrals of the second momentum of the GPDs).

where $q^2 = -t$ and Λ_1 and Λ_2 are determined by fitting the obtained value of $A_{Gr,t}(t)$. However, firstly it is necessary to examine the different PDF sets obtained by the different collaborations from the analysis of the dip inelastic scattering data to see which PDFs better describe the electromagnetic form factors.

5 The pp and $p\bar{p}$ elastic differential cross sections at high energies

The differential cross sections of the nucleon-nucleon elastic scattering can be written as the sum of the different spiral scattering amplitudes: Every amplitude $\phi_i(s, t)$, including the electromagnetic and hadronic forces, can be expressed as

$$\phi(s, t) = F_C \exp(i\alpha\varphi(s, t)) + F_N(s, t), \quad (16)$$

with $\varphi(s, t) = \varphi(t)_C - \varphi(s, t)_{CN}$, where $\varphi(t)_C$ appears in the second Born approximation of the pure Coulomb amplitude, and the term φ_{CN} is defined by the Coulomb-hadron interference [26,27,28].

In [29], it was proposed that the hadron scattering amplitude includes two parts which are proportional to two forms of the form factors. One form factor is related to the charge distribution to the hadron and represented in the standard form of the electromagnetic form factor - $G(t)$. The second form factor is proportional to the momentum energy tensor distribution and can be represented by the gravitational form factor - $A(t)$. Both the forms of the form factors can be obtained from the same GPDs as first and second moments.

The hadron Born term of the elastic nucleon amplitude can be written as

$$F_N^{\text{Born}}(s, t) = h_1 G^2(t) F_1(s, t) (1 + R_1/\hat{s}^{0.5}) + h_2 A^2(t) F_2(s, t) (1 + R_2/\hat{s}^{0.5}) \quad (17)$$

where $F_a(s, t)$ and $F_b(s, t)$ has the standard Regge form

$$F_1(s, t) = \hat{s}^{\epsilon_1} e^{B(s)t}; \quad F_2(s, t) = \hat{s}^{\epsilon_1} e^{B(s)/4 t}, \quad (18)$$

with $G(t)$ being the electromagnetic form factor relative to the first moment of GPDs and $A(t)$ relative to the second moment of GPDs.

$$G(t) = \frac{L_1^4}{(L_1^2 - t)^2} \frac{4m_p^2 - (1+k)t}{4m_p^2 - t}; \quad A(t) = \frac{L_2^4}{(L_2^2 - t)^2}; \quad (19)$$

with the parameters: $L_1^2 = 0.71 \text{ GeV}^2$; $L_2^2 = 2 \text{ GeV}^2$. The values of R_1 and R_2 are taken as free parameters which reflect the rest of the low energy contributions to the high energy parts. Using the crossing symmetry properties of the scattering amplitudes of the proton-proton and proton-antiproton scattering at high energies we take $\hat{s} = s e^{-i\pi/2}/s_0$; with $s_0 = 1 \text{ GeV}^2$. In this case, the real part of the scattering amplitude has no any additional parameters. The slope of the

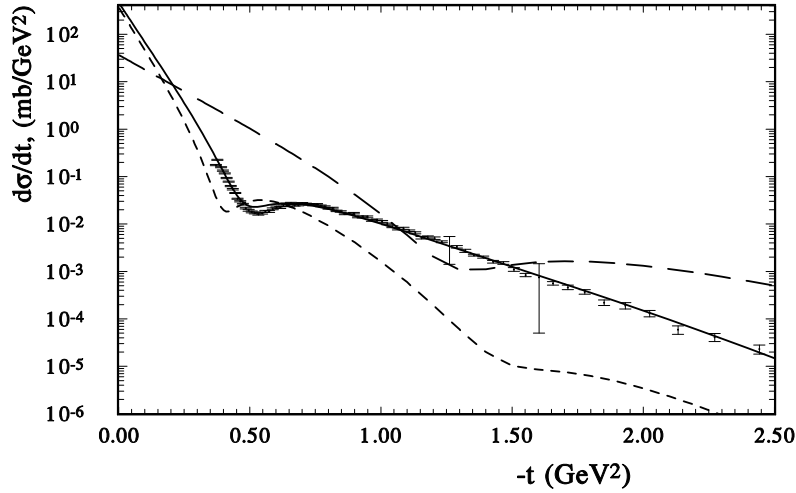


Fig. 3. The differential cross sections of the pp elastic scattering at TeV. The hard line is the model predictions; the short dashed and long dashed lines present the differential cross section in the case if we take into account the part of the scattering amplitude with the electromagnetic form factor or the gravitational form factors.

scattering amplitude has the standard logarithmic dependence on the energy. $B(s) = \alpha' \ln(\hat{s})$ with $\alpha' = 0.24 \text{ GeV}^{-2}$. In the model, the Born terms of the scattering amplitude are calculated only. Then the final scattering amplitude is obtained through the standard eikonalization procedure [30,31]. First, we calculate the eikonal phase in the impact parameter representation and then, after integrating over the impact parameter, we obtain the final scattering amplitude. In Fig. 3, the parts of the differential scattering corresponding to the Born parts of the scattering amplitude are shown. Of course, the final result is not the simple sum of the separate parts determined by the separate Born terms. Only the eikonalization of the Born amplitude leads to the good representation of the experimental differential cross sections. In Fig. 3, we can see that the first term of the scattering amplitude, proportional to the electromagnetic form factor, determines the differential cross sections at small momentum transfer. At large momentum transfer the second term of the Born amplitude, proportional to the gravitational form factors, gives the dominant contribution.

As a result, we can obtain a good description of the existing differential cross sections of the proton-proton and proton-antiproton at high energies [29], and the model prediction coincides with the new TOTEM data at 7 TeV. In Fig. 3, the predictions of the model for the 7 TeV are shown. Except the size of the diffraction minimum the coincidence of the model and experimental data is very good.

6 Conclusion

The new data of the TOTEM Collaboration [1,2] presented the behavior of the differential cross sections at small and sufficiently large momentum transfer (up to $-t = 2.5 \text{ GeV}^2$). These results do not coincide with the predictions of the most popular models of hadron interactions at high energies. We show that the using two form factors - the electromagnetic and the gravitational form factors, which are obtained from the same General Parton Distributions as first and second moments respectively, we can obtain the description of all high energy elastic scattering data in small and large momentum transfer regions. This opens the new way for connecting the deep inelastic scattering processes, from which the parton distributions were obtained, with the elastic hadron scattering.

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